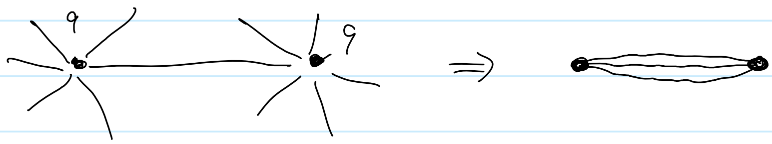
The String's The Thing

$10^{-15} \to 10^{-34}$

String Theory backed quite unintentionally into this story.

Trying to explain the absence of free quarks in the strong interactions, folks considered flux tubes:



Field lines go all over
$\Rightarrow$ force weakens w/ distance

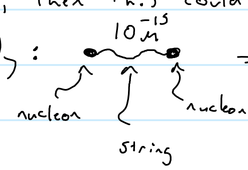
Field lines confined to flux tube
$\Rightarrow$ force strengthens w/ distance

In studying the quantum dynamics of these flux tubes, physicists were perplexed by a few things they found:

1) They only seemed to make sense in $D > 4$ dimensions
2) They had tachyonic excitations
3) The spectrum included an annoying spin-2 particle

All of these were problematic for a theory only trying to describe the strong nuclear force ($l_s \sim 10^{-15} m$)

But it was realized that if instead we let $l_s \sim 10^{-34} m$, then this could actually be a TOE where the spin-2 particle is the graviton! Note originally:

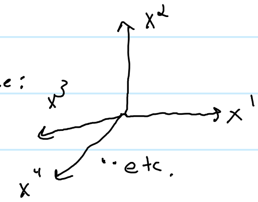


$\Rightarrow$ String Theory $\sim 10^{-34} m$
fund. "particle"

The claims above are all pretty remarkable, so let's look at each a little more carefully.

To begin, how does one figure out how many dimensions string theory lives in, or rather how does one even start to do physics w/out a specified # of dimensions.

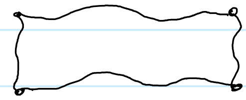
Spacetime:



Point Particle

$X^\mu(\tau)$

String

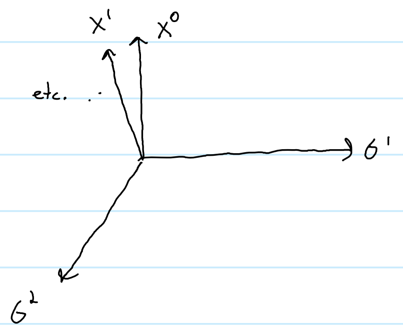


$X^\mu(\sigma^1, \sigma^2)$

So we think of τ or (σ^1, σ^2) as parameterizing worldvolume and in principle we would solve some spacetime e.o.m. for $X^\mu(\tau)$ or $X^\mu(\sigma^1, \sigma^2)$.
Note: These are <u>not</u> field theories!

On the other hand, if we think about what we are looking for, we realize that the d.o.f. could alternately described in terms of field theories on a space coordinatized by τ or (σ^1, σ^2).

Worldsheet: etc. . .



Now we view X^μ as a collection of (embedding) fields which are worldsheet <u>scalars</u>.

What 2D action would control this?

For the point particle, we know that a free particle moving through spacetime will follow a geodesic path which extremizes its length (actually max in Lorentz signature).

$$S_{pp} \propto \int ds = \int \sqrt{ds^2} = \int \sqrt{dx_\mu dx^\mu}$$
$$= \int d\tau \sqrt{\frac{dx_\mu}{d\tau}\frac{dx^\mu}{d\tau}}$$

which is reparameterization invariant, e.g. $\tau \to \tau'(\tau)$

But we also know that the result is different for $m \neq 0$ (for which $ds^2 < 0$) and $m=0$ ($ds^2=0$). To incorporate this:

$$S_{pp} = -m \int d\tau \sqrt{\frac{dx_\mu}{d\tau}\frac{dx^\mu}{d\tau}}$$
$$= \frac{1}{2}\int d\tau \left(\frac{1}{\gamma}\frac{dx_\mu}{d\tau}\frac{dx^\mu}{d\tau} + \gamma m^2\right)$$

where $\gamma_{\tau\tau}(\tau)$ is the worldline metric which has e.o.m. $\gamma^2 = -\frac{\partial x_\mu}{\partial\tau}\frac{\partial x^\mu}{\partial\tau}/m^2$

Note, this makes sense even w/ $m=0$!

Now we just generalize this to 2D: $S_{NG} = -T \int d\sigma^1 d\sigma^2 \sqrt{-\underbrace{\det\left(\frac{\partial x_\mu}{\partial\sigma^a}\frac{\partial x^\mu}{\partial\sigma^b}\right)}_{\text{reduces to } \frac{\partial x_\mu}{\partial\tau}\frac{\partial x^\mu}{\partial\tau} \text{ for only 1 coord.}}}$ $\quad a,b \in 1,2$

T: tension

This can be extended w/ worldsheet metric γ_{ab} to: $S_p = -\frac{T}{2}\int d\sigma^1 d\sigma^2 \sqrt{-\det\gamma}\,\gamma^{ab}\frac{\partial x^\mu}{\partial\sigma^a}\frac{\partial x_\mu}{\partial\sigma^b}$

Now so far our worldsheet theory only contains D - X^μ fields and γ - metric field.
This is enough to formulate what is called the "bosonic string", that is a string theory with only bosonic d.o.f.
But our world has fermions. So to extend string theory to accomodate these, we will introduce another type of field to the worldsheet theory, ψ^μ, which is a worldsheet spinor. Now to do this in a clean way we introduce it w/ worldsheet supersymmetry, i.e. a symmetry transformation which interchanges all bosons $\leftrightarrow$ fermions, but does not change S.
But to interchange X^μ w/ ψ^μ, the μ-indices must play the same role. So ψ^μ is a spacetime vector.

But there is something else we can and should add to the worldsheet theory.
Notice that $X^\mu(\sigma^1, \sigma^2)$ will be describing the D-dim "wiggling" of the string surface in spacetime.

The index μ runs over all of spacetime, but there are always two directions of "wiggling" that are non-physical... the two along the strings worldsheet. We could ignore these (go to light-cone gauge) or we can cancel these oscillations with another set of fields called Fadeev-Popov "ghosts". So to cancel these 2 X^μ-oscillations we add b, c ghost fields. To do the same for ψ^μ oscillations we add β, γ ghost fields.

So in all our theory contains $\{\gamma, D\,X^\mu, D\,\psi^\mu, b, c, \beta, \gamma\}$

Going back to the simplest action:

$$S_p = -\frac{T}{2}\int d\sigma^1 d\sigma^2 \sqrt{-\det\gamma}\;\gamma^{ab}\frac{\partial X^\mu}{\partial\sigma^a}\frac{\partial X_\mu}{\partial\sigma^b}$$

Brink, Di Vecchia, Howe, Desser, Zumino action or Polyakov for short.

This has some very important symmetries (as do the the theories w/ fermions and ghosts):

1) Poincaré invariance rotating fields into each other (an internal symmetry)

2) 2D diff $X'^\mu(\sigma'^1,\sigma'^2) = X^\mu(\sigma^1,\sigma^2)$, $\gamma_{a'b'} = \frac{\partial\sigma^a}{\partial\sigma^{a'}}\frac{\partial\sigma^b}{\partial\sigma^{b'}}\gamma_{ab}$ — 2

3) Weyl inv. $X'^\mu(\sigma^1,\sigma^2) = X^\mu(\sigma^1,\sigma^2)$, $\gamma_{a'b'} = e^{2\omega(\sigma^1,\sigma^2)}\gamma_{ab}$ — 1

In arbitrary D: D, 1

$\underbrace{\text{only works in } D=2!!}$

The last 3 symmetries can be exploited to fix $\gamma_{ab} = \begin{pmatrix} a & b \\ b & c\end{pmatrix}$ to be $\gamma_{ab} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$, so our worldsheet is now flat, and we can quantize the $X^\mu(\sigma^1,\sigma^2)$ field theory as usual.

However whenever we use symmetries in the classical theory to simplify things for quantization, we should always check whether the symmetries survive in the quantum version, i.e. is the path integral $PI = \int dX e^{-S_{cl}}$ invariant? We know S_{cl} is so we are really looking at dX.

One finds that 2D diff. holds, but Weyl inv. produces an anomaly, i.e. $\delta_w PI \propto C$

central charge of the 2D QFT

In fact each field on our theory contributes:

Field	Contribution to C
Each X^μ	$+1$
Each ψ^μ	$\frac{1}{2}$
b, c	-26
β, γ	11

To get rid of the anomaly we need:

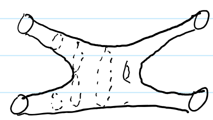
$$\begin{cases} \text{Bosonic } (X^\mu, b, c) & D-26=0 \Rightarrow D=26 \\ \text{Boson/Fermions } (X^\mu, b, c, \psi^\mu, \beta, \gamma) & D-26+\frac{D}{2}+11=0 \Rightarrow D=10\end{cases}$$

Now that we have content, let's talk about shape.

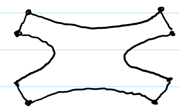
<u>2 types</u>

$\Rightarrow$

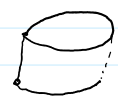
<u>Only Closed</u>



<u>Only ~~Closed~~</u>



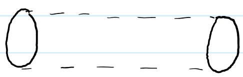
but...



open string loop
or
closed string propagator

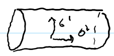
so rather Open + Closed

We'll focus on closed strings for now. We would like to know the spectrum as seen in spacetime.



We decorate this w/ the ground state of all fields + excitations.
Now it turns out that the left/right (counter/clockwise) d.o.f. are independent (can't be cont. rotated into)
So it turns out that we three possible constructions:

$L(26X^\mu, b, c)\,R(26X^\mu, b, c) \Rightarrow$ 26D bosonic string
$L(10X^\mu, b, c, 10\psi^\mu, \beta, \gamma)\,R(10X^\mu, b, c, 10\psi^\mu, \beta, \gamma) \Rightarrow$ Type II strings
but also
$L(10X^\mu, b, c, 10\psi^\mu, \beta, \gamma)\,R(10X^\mu + \underbrace{16\lambda^i}_{i \text{ is an internal index so corresponds to gauge symmetry}}, b, c) \Rightarrow$ 10D heterotic strings w/ rank 16 $SO(32)$ or $E8\times E8$

Now to discern the spectrum, we need to know the energy/tension/mass of each ground (for all fields on the worldsheet) then also how they transform in spacetime. But there is a freedom.
When we take X^μ around the worldsheet, i.e.  $X^\mu(\sigma^1+2\pi, \sigma^2) = X^\mu(\sigma^1, \sigma^2)$ since X^μ is intimately tied to spacetime.

But for ψ^μ we have freedom in choosing the spin-structure: $\psi^\mu(\sigma^1+2\pi, \sigma^2) = \psi^\mu(\sigma^1, \sigma^2)$ Ramond $\Rightarrow$ ground state is a spinor
$\psi^\mu(\sigma^1+2\pi, \sigma^2) = -\psi^\mu(\sigma^1, \sigma^2)$ Neveu-Schwarz $\Rightarrow$ ground state is scalar

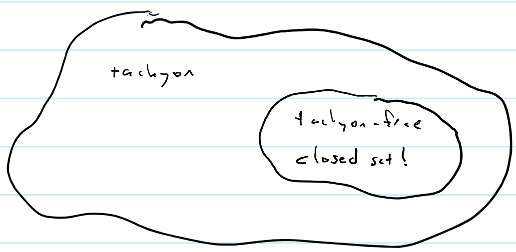
So in putting the two sides together:

L	R		<u>mass (m^2)</u>
NS	- NS	$\Rightarrow$ scalar	$-\frac{2}{\alpha'}$
R	- R	$\Rightarrow$ p-form	0
R	- NS	$\Rightarrow$ spinor	0
NS	- R	$\Rightarrow$ spinor	0

zero modes

we can bring this to $m=0$ by exciting each w/ $\underbrace{\tilde{\psi}^\mu \psi^\nu}_{H^{\mu\nu} = \text{scalar} + \text{anti} + \underbrace{\text{symm}}_{g^{\mu\nu} - \text{graviton}}}|0\rangle_L|0\rangle_R$

What about that tachyon? Doesn't it imply an instability? Yes it does... but we can project.



Gliozzi, Scherk and Olive (GSO) projection.

Leads to spacetime supersymmetry!

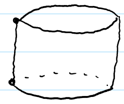
What about that extra six dimensions? One answer ... compactification, i.e. $M^{10} \to M^4 + T^6$

Can we see them? Well recall from QM that $p_{S^1_R} \propto \frac{n}{R}$, so if $R \to$ super small $\Rightarrow$ $p_1 \to$ super huge (and so too $n > 1$)
thus only p_0 is possible.

But this opens the door to dualities. Consider S^1_R w/ $E = p_{S^1_R} = \frac{n}{R}$. If I take $R \to R^{-1}$ then this spectrum changes $E' = nR$.
But that is for particles. These are strings. In particular a string can wrap itself around S^1_R giving a contribution to E of mR.
Thus: $E = \frac{n}{R} + mR \xrightarrow{R \to R^{-1}} nR + \frac{m}{R} = E$, i.e. same energy spectrum $\Rightarrow$ T-duality ($R \to \frac{1}{R}$). Relates bosonic $\leftrightarrow$ bosonic, IIA $\leftrightarrow$ IIB.

Does string theory cure the ∞'s from quantum gravity?

Infinities come from virtual particles:  $\Rightarrow \int_0^\infty dq \Rightarrow$ higher q or smaller circle $\Rightarrow$ divergence

What about strings?  now take $R \to 0$, this can then be interpreted as  long distance!

or for closed 

taking $R \to 0$ is impossible due to modular invariance!